

ELEC50001 EE2 Circuits and Systems

Problem Sheet 5 Solutions – Data Converters (Lectures 10 and 13)

(Question ratings: A=Easy, ..., E=Hard. All students should do questions rated A, B or C as a minimum)

- 1B. Full-scale range = 3998 so the accuracy is $2/3998$ of full-scale range. For an N-bit binary A/D converter, the full-scale range is $(2^N - 1)$ LSB giving an accuracy of $0.5/(2^N - 1)$.

$$\text{Hence } \frac{0.5}{2^N - 1} \leq \frac{2}{3998} \Rightarrow 2^N \geq 1000.5 \Rightarrow N \geq 9.96 \Rightarrow N = 10$$

- 2B. $-2047 \text{ mV} \pm 0.5 \text{ mV}$, i.e. -2047.5 mV to -2046.5 mV .

- 3B. 1 V and 8 V correspond to output values of 100 and 800 respectively, so if $1 \text{ V} < x < 8 \text{ V}$, the output will be in the range 100 to 799.

- 4C. A change of 5 V in V_3 must give a change of -1 V in V_{OUT} , a gain of -0.2 . Hence $R_F/R_3 = 0.2 \Rightarrow R_3 = 50 \text{ k}\Omega$.

When $V_3=0$, the op-amp may be viewed as a non-inverting amplifier with a gain of $(1 + R_F/R_3) = 1.2$. The voltage at V_{OUT} due to $V_2=0$ is therefore given by:

$$V_{OUT} = 1.2 \times \frac{G_2 V_2 + G_1 V_1 + G_0 V_0}{G_4 + G_2 + G_1 + G_0}$$

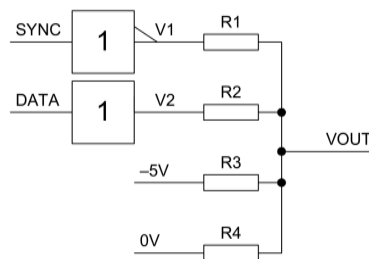
where $G_4=0$ are the reciprocals of $R_4=0$.

To minimize the effect of op-amp bias currents, we should make the Thévenin impedances at the input terminals equal. This means that $G_4 + G_2 + G_1 + G_0 = G_3 + G_F = 120 \mu\text{S}$.

The gains from V_2 , V_1 and V_0 to V_{OUT} must be 0.1, 0.05 and 0.025 respectively. Thus we have $G_2 = 120 \mu\text{S} \times 0.1/1.2 = 10 \mu\text{S} \Rightarrow R_2 = 100 \text{ k}\Omega$. Similarly, $R_1 = 200 \text{ k}\Omega$ and $R_0 = 400 \text{ k}\Omega$.

Finally $G_4 = 120 \mu\text{S} - G_2 - G_1 - G_0 = 102.5 \mu\text{S} \Rightarrow R_4 = 9.8 \text{ k}\Omega$.

- 5C. The SYNC signal needs inverting because SYNC going high must cause the output to decrease. We will need a negative bias voltage in order to obtain -0.3 V . Our circuit is therefore:



Taking $G_n = 1/R_n$ we must have $G_1 + G_2 + G_3 + G_4 = 1/50 \Omega = 20 \text{ mS}$.

Then $V_{OUT} = (V_1 G_1 + V_2 G_2 - 5 G_3) / 20 \text{ mS}$.

From the truth table, we see that changes of 5 V in V_1 and V_2 must give changes in V_{OUT} of 0.3 and 0.7 volts respectively; this means we need gains of 0.06 and 0.14. Hence:

$$G1 = 0.06 \times 20 \text{ mS} = 1.2 \text{ mS} \Rightarrow R1 = 833\Omega.$$

$$G2 = 0.14 \times 20 \text{ mS} = 2.8 \text{ mS} \Rightarrow R2 = 357\Omega.$$

$$\text{To generate the } -0.3 \text{ V offset: } 5G3 = 0.3 \times 20 \text{ mS} = 6 \text{ mS} \Rightarrow R3 = 833\Omega.$$

$$G4 = 20 \text{ mS} - G1 - G2 - G3 = 14.8 \text{ mS} \Rightarrow R4 = 67.6\Omega.$$

Note it is possible to take R4 to +5 V instead in which case R3 and R4 are 116 Ω and 135 Ω .

This circuit is very fast since it has no op-amps.

6B. The range of a 16-bit signed number is ± 32767 and so to avoid distortion, the RMS value must be no higher than 3276.7. From the notes, the RMS value of quantisation noise is 0.289 LSB which gives a signal-to-noise ratio of 11338 which equals 81 dB

7C. Full-scale range of 20 V equals 4096 LSB so $0.5 \text{ LSB} = 0.5 \times 20/4096 = 2.44 \text{ mV}$.

The peak rate of change of a 10 V sinewave is $20\pi f$ volts per second. The voltage change in 5 ns is therefore $\pi f \times 10^{-7}$. These are equal when $f = 2.44 \times 10^{-3} \times 10^7 / \pi = 7.77 \text{ kHz}$.

For the second part $I = C \text{ dV/dt}$ from which $\Delta t = C \times \Delta V / I = 2 \times 10^{-10} \times 2.44 \times 10^{-3} / 10^{-9} = 488 \text{ } \mu\text{s}$.